Math Solution
...a power series solution to certain differential equation

$y = ce^x$

→ Lagranges linear equation → $Pp + Qq = R$.
where P, Q, R are functions of x, y, z, which is of 1st order & linear in p & q.

e.g. $(y+z)p + (z+x)q = x+y$ is lagranges linear equation

where $p = \frac{\partial z}{\partial x}$ $q = \frac{\partial z}{\partial y}$

* Subsidiary / auxiliary eq^n.

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

$f(u,v) = 0$, $u = f(v)$ or $v = f(u)$

* Homogenous linear eq^n with constant co-efficients

$$P(t)y'' + Q(t)y' + R(t)y = G(t). \quad \boxed{y'' + P(t)y' + q(t)y = g(t)}$$

$$ay'' + by' + cy = 0$$

* Non homogenous

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = r(x)$$

* Application of partial differential eq^n

1. Solution of wave eq^n → $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ $\qquad \frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$
2. // // Heat //
3. // // laplace //

$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ ch

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$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ $a\lambda t$

suitable solution eq^n

k +ve & $k = \lambda^2$

$y = (c_1 e^{\lambda x} + c_2 e^{-\lambda x})(c_3 e^{a\lambda t} + c_4 e^{-a\lambda t})$

$k =$ -ve $k = -\lambda^2$

$y = (c_5 \cos \lambda x + c_6 \sin \lambda x)(c_7 \cos a\lambda t + c_8 \sin a\lambda t)$

$k = 0$

$y = (c_9 x + c_{10})(c_{11} t + c_{12})$

2. Solution of heat eqn — one dimensional heat equation

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

K +ve & $K = \lambda^2$

$$u = (c_1 e^{\lambda x} + c_2 e^{-\lambda x}) c_3 e^{a^2 \lambda^2 t}$$

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

K -ve & $K = -\lambda^2$

$$\boxed{u = (c_4 \cos \lambda x + c_5 \sin \lambda x) c_6 e^{-a^2 \lambda^2 t}}$$

$\downarrow$ suitable heat solution

$$\frac{\partial^2 u}{\partial x^2} + \partial^2$$

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

K = 0

$$u = (c_7 x + c_8) c_9$$

$\partial^2 u$

3. Solution of Laplace's eqn (Two dimensional heat eqn)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2}$$

$$u = (c_1 e^{\lambda x} + c_2 e^{-\lambda x})(c_3 \cos \lambda y + c_4 \sin \lambda y)$$

$$u = (c_5 \cos \lambda x + c_6 \sin \lambda x)(c_7 e^{\lambda y} + c_8 e^{-\lambda y})$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$u = (c_9 x + c_{10})(c_{11} x + c_{12})$$

4. Heat flow eqn. $\longrightarrow$ 1 dimension

$$Q = c \times m \times \Delta T \qquad \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Solution by method of seperation of variables.

assumption & seperable $\to$ find solution $\to$ represent $\to$ product of several functions each of which is only dependent upon a single independent variable

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$$

$CSA = \pi r l$
$= \frac{22}{7} \times 10 \times 20$

628.57 cm^2

$T = H/K$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

T.S.A. $= \pi (r + l) r$
$= \frac{22}{7}(10 + 20) 10$
$= \frac{22}{7} (30 \times 10$
$= \frac{22}{7}$

$20/\lambda$

$$\frac{\partial u}{\partial t} = a^2/c^2 \frac{\partial^2 u}{\partial x^2}$$

$\frac{1}{3} \pi r^2 h$

$\frac{1}{3} \frac{22}{7} \frac{20}{20}$